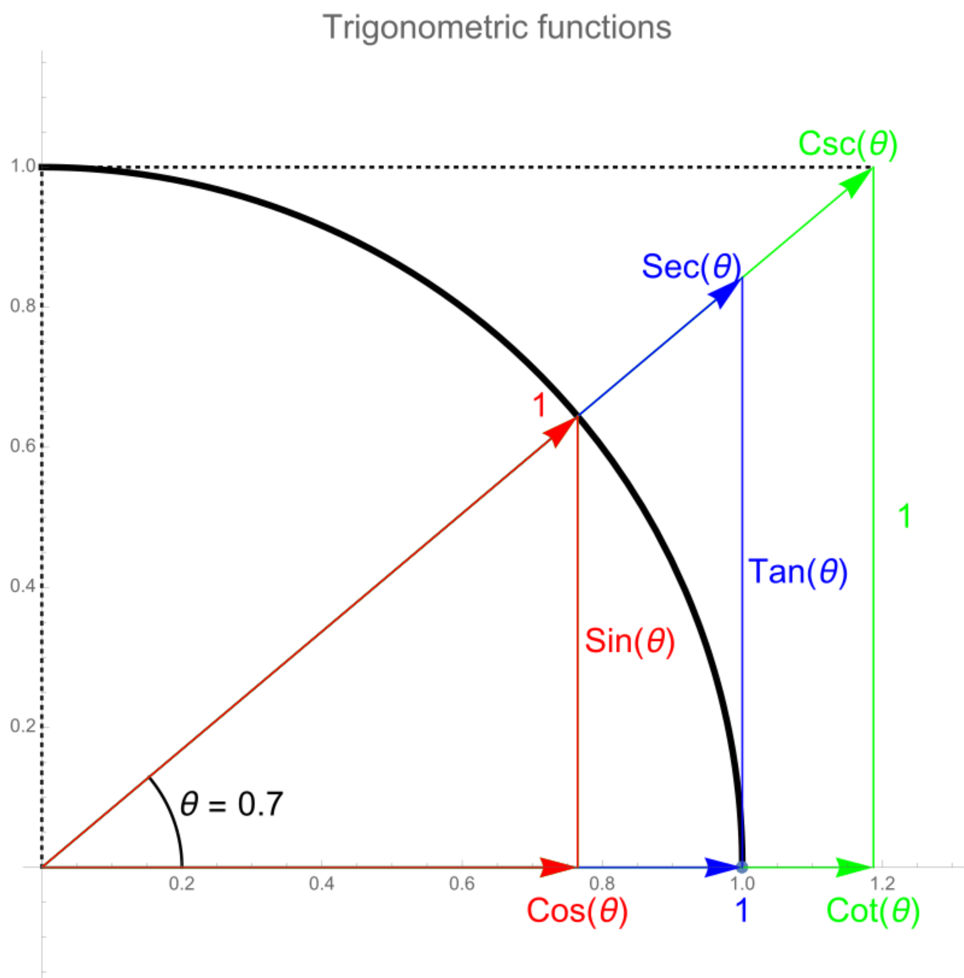
Suppose you draw the line x=1 on a unit circle. That line is tangent to the unit circle at (1,0). The tangent of an angle is the length of the line segment from (1,0) to wherever the angle intersects the line x=1. That is, the distance from (1,0) to (1, tan θ). *That's* why it's called the "tangent" of the angle. Simple, really. But how many trig instructors bother to mention this? For that matter, how many know it?

So why do we say the tangent is the sine/cosine? If you look at the diagram, it's apparent that we're talking about similar triangles, so sine/cosine = tangent/1.



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Let's go further. Few people have any idea what a secant is (again including me, until quite recently), but it's simple. Extending an angle from (0,0) to the intersection with the tangent line gives a line segment called the "secant," derived from the Latin verb "to cut," because it cuts the unit circle. A glance at the diagram and the Pythagorean theorem reveals that $\tan^2\theta + 1^2 = \sec^2\theta$, an otherwise obscure proposition to most people.

At the risk putting everyone into a coma, let's finish this up. Note that the cosine is the horizontal conjugate of the sine. The "co-" prefix indicates the horizontal conjugate of a vertical quantity.

So what is the cotangent?* It's where an angle intersection the *horizontal* line y =1. And the cosecant is the length of the resulting line segment.

Now, consider what we've done. In five minutes we've laid out trigonometry, which normally requires a year-long course, and we've done it in an intuitive way that is easily remembered and used. Suppose we want to know the height of a tree. From our diagram it is apparent that if we're one meter away from the tree, the tangent of the angle from the ground to the top of the tree gives us its height. If we're N meters away, we just multiply that tangent by N. Simple, and much more intuitively apparent than messing with sines and cosines (which give us the same result, but in a more obscure fashion).

In similar fashion, I never really understood where the Lagrange multiplier came from. Yeah, I could do the algebra, but did not understand where it had come from. Whatever led Lagrange to come up with this, anyway? From a geometric perspective it is simple, almost obvious. The shortest distance from curve *f* to curve *g* will lie a normal from *f*, i.e., the vector ▼*f* (sorry, closest thing I could find to the del operator symbol in MS Word). Of course, the same is true of g. Since the shortest distance from f to g is the same as that from g to f, the two normals will lie on the same line. But there's no necessary relationship between the magnitude of the normals, nor even their direction, so ▼*f* = λ▼*g*, where λ is a scalar (i.e., just a number). They differ from each other by - say it with me - an undetermined multiplier.

Voila!

* This diagram indicates the cotangent along the x-axis; for choice I would prefer it being indicated along the line y =1, to maintain symmetry and to make it obvious where it comes from.